Forms of Reinsurance

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Abstract

Reinsurance plays an important role in reducing the risk in an insurance portfolio. In this article we give an overview of the most relevant classical reinsurance concepts from a practical point of view. We will explain how the different forms of reinsurance affect the stability of insurance portfolios.

Keywords: solvency margin, dependence, ruin probability paradigm, quota-share treaty, surplus treaty, excess-of-loss treaty, adjustment coefficient

The law of large numbers is seen as the basis of modern insurance because for homogeneous groups the average loss will converge to the expected loss when the size n of the group becomes larger, i.e. $\sum_j^n X_j/n \to \mu$. Hence for estimating possible losses and rating purposes, the larger the group the more predictive power is reached. However for solvency requirements of the total portfolio the total claim size $\sum_j^n X_j$ is the relevant variable. In order to explain the tail danger due to dependencies (e.g. when different floors of one building are insured against fire) we consider the case of identically distributed random variables. Its standard deviation in the independent case then reads $\sqrt{n}\sigma$ but in the comonotone case it equals $n\sigma$. This shows that for the solvency position of an insurance company the calculated solvency margin (e.g. as a **value at risk** (VaR)) increases with $\sqrt{n}$ or n as indicated in [2]. This puts a heavy burden on the solvency margin required in branches where the risks might be dependent. Note that stochastic models used for the rate of return of cash flows (resulting from the development of reserves) will impose a dependence structure on the terms in the sum of the stochastic capital at risk too, which might result in relatively high solvency margins. This situation might lead to an unexpected difference between the predicted and the realized value. For this situation reinsurance plays an important role. The forms of reinsurance are designed in such a way that the remaining risk in a portfolio decreases. Hence in order to optimize a reinsurance policy, a trade-off between profitability and safety will be the key issue, see e.g. [1].

All types of reinsurance contracts aim at several goals in the framework of risk reduction, stability of the results, protection against catastrophes, increase of underwriting capacity, etc. The main criterion used is the so-called *ruin probability paradigm*.

Two types of reinsurance exist: facultative reinsurance and obligatory reinsurance. Facultative reinsurance is a case-by-case reinsurance where each individual risk before acceptance, and exceeding the retention of the direct insurer, is presented to the reinsurer. Both the direct insurer and the potential reinsurer are free to present or accept the risk. Facultative reinsurance is tailor-made for each application of insurance. An intermediate case is the so-called open cover. In the case of obligatory reinsurance, every claim within the specifications of the reinsurance treaty is ceded and accepted by the reinsurer.

The **quota-share** treaty is a treaty between the ceding company and the reinsurer to share premiums and claims with the same proportion. When the individual insurance contract insures a risk X for a premium $\pi(X)$, the ceding company and the reinsurer will divide the risk X and the premium $\pi(X)$ as $(pX, p\pi(X))$ and $((1-p)X, (1-p)\pi(X))$. Of course this doesn't imply $\pi(pX) = p\pi(X)$. The reinsurance treaty can be considered as an economic deal. Because the ceding company organizes the selling, acquisition, pricing and the claims handling of the contracts, the reinsurer pays a ceding commission to the ceding company, see e.g. [3].

In a **surplus treaty** the reinsurer agrees to accept an individual risk with sum insured in excess of the direct retention limit set by the ceding company (expressed in monetary units). This treaty subdivides the risk in lines. A first line, e.g. [∈0,∈100,000] providing a retention limit of ∈100,000 to the ceding company, is augmented with e.g. 3 lines such that the insurance capacity is increased up to ∈400,000. The ceding company retains the lines above ∈400,000. The ceding company and the reinsurer share the premiums and the claims in a proportional way where the proportion is calculated as follows. Suppose the sum insured amounts to ∈300,000. The ceding company then retains ∈100,000 or one third and the reinsurer gets ∈200,000 or two thirds of the amount to reinsure. This determines the proportion $(1/3, 2/3)$ of earned premiums and shared claims on a proportional basis. Also in this case the reinsurer pays a ceding commission to the ceding company.

In the **excess-of-loss** treaty - XL - (see **Excess of loss**) a retention limit d is considered. The cedent then retains the risk $X-(X-d)_+ + [X-(d+u)]_+$ while the reinsurer intervenes for the amount $\min[(X-d)_+, u]$. This principle may be applied to a single exposure (the so-called WXL-R or Working XL per risk, where an individual claim insured can trigger the cover) or a single occurrence such as a storm risk (the so-called CAT-XL or **Catastrophe XL**, where a loss event involves several covered individual risks at the same time). In this case there is no direct relation between the premium of the risk X and the price of the reinsurance of $\min[(X-d)_+, u]$. Premiums can be calculated according to different rules. The **stop-loss** treaty is technically equivalent to an excess-of-loss treaty but the risk considered now is the total claim size of a portfolio as is explained in [4].

More advanced forms of reinsurance include the *Excédent du coût moyen relatif* treaty (ECOMOR) and the Largest Claim treaty (LC). In some sense the ECOMOR(p) treaty rephrases the excess-of-loss treaty but with a random retention at a large claim, since it covers all claims excess the pth largest claim. The LC(p) treaty also focusses on large claims by covering the p largest claims.

Within a reinsurance **pool** a group of insurers organize themselves to underwrite insurance on a joint basis. Reinsurance pools are generally considered for specific catastrophic risks such as aviation insurance and nuclear energy exposures. The claims are distributed among the members of the pool in a proportional way.

One of the main results of **utility theory** in the framework of reinsurance forms, limited to be a function of the individual claims, is that the excess-of-loss contract is optimal in case the expectation is used as the pricing mechanism. Indeed, among all possible reinsurance contracts $h(x)$ with $0 < h(x) < 1$ and $\mathrm{E}[h(X)]$ fixed, the excess-of-loss contract $h(x) = (x-d)_+$ maximizes utility.

References

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